The experimental design is a statistical method to analyze which factors are affecting the results of the target. The cost of collecting data for the analysis is still a big problem, especially in the medical and healthcare fields. In this paper, we propose simultaneous experiments for related linear models based on an orthonormal system. The cost can be reduced by the simultaneous experiments.

**Previous studies**

- $\mathcal{M}_1 \rightarrow $ Data
- $\mathcal{M}_2 \rightarrow $ Data
- $\mathcal{M}_3 \rightarrow $ Data

The cost for data collection is minimized for each model.

**This study**

$\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3 \rightarrow $ Data for $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$

We minimize the total sum of individual costs.

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**Example (Models of 2 Factors with 3 Levels)**

**Conventional Model**

$t(x) = \mu + \alpha_1(x_1) + \alpha_2(x_2) + \epsilon$, where
- $\mu$: the general mean (GM)
- $\alpha_1(x_1)$: the effect of $x_1$ at level of $F_1$
- $\alpha_2(x_2)$: the effect of $x_2$ at level of $F_2$
- $\beta_{12}(x_1, x_2)$: the interaction of $x_1$ and $x_2$ of $F_1$ and $F_2$
- $\epsilon$: a zero-mean Gaussian random variable with $\sigma^2$

**Notations**

- $F_1, F_2, \ldots, F_m$: the factors
- $x_i \in \{0, 1, \ldots, q\}$: the level of $F_i$

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**Bayesian Experimental Designs**

**Likelihood Function**

The likelihood function $p(t|X, u, \sigma^2)$ can be expressed by

$p(t|X, u, \sigma^2) = N(t|\Phi w, \sigma^2 I)$,

where

$p(t|X, u, \sigma^2)$

- $\Phi$: a vector that contains all factors of interest in $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_L$

**Prior and Posterior Probability**

- The corresponding conjugate prior is given by $p(u) = N(u|m_0, S_0)$,
- The posterior probability is given by $p(u|X, t, \sigma^2) = N(u|m_n, S_n)$,

where

$S_n = S_0 \left( \frac{1}{\sigma^2} (M^{-1})^T \Phi^T X + S_0^{-1} \right)$

**Bayesian experimental designs**

- A-optimality: Minimize trace $[S_N]$
- D-optimality: Maximize det $[S_N^{-1}]$

In this work, we focus on the design that satisfies $\frac{1}{\sigma^2} \Phi^T \Phi = I$.

**Posterior variance for orthogonal designs**

$S_N = \left( \frac{N}{\sigma^2} (M^{-1})^T + S_0^{-1} \right)^{-1}$

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**Conclusions**

- We've proposed simultaneous experiments for related linear models based on an orthonormal system. The cost can be reduced by the simultaneous experiments.
- We've shown the posterior variance for orthogonal designs.